$\therefore y = e = 1$ Ans.

(50) Find the Limit of $(\cot x)^{\frac{1}{\log x}}$ at $x = 0$

Ans. → $\lim\limits_{x \to 0} (\cot x)^{\frac{1}{\log x}}$ [

Let $y = \lim\limits_{x \to 0} (\cot x)^{\frac{1}{\log x}} \left[\infty^0\right]$

Taking log both sides, we have,

$$\log_e^y = \lim_{x \to 0} \frac{1}{\log x} \cdot \log \cot x$$

$$= \lim_{x \to 0} \frac{\log \cot x}{\log x} \left[\frac{\infty}{\infty}\right]$$

Hence, from L' Hospital's Rule, we have,

$$\log_e^y = \lim_{x \to 0} \frac{\frac{-\operatorname{cosec}^2 x}{\cot x}}{\frac{1}{x}}$$

$$= \lim_{x \to 0} \frac{\frac{-\frac{1}{\cancel{\sin^2 x}\,\sin x}}{\frac{\cos x}{\cancel{\sin x}}}}{\frac{1}{x}}$$

$$= \lim_{x \to 0} \frac{-x}{\sin x \cdot \cos x} \left[\frac{0}{0}\right]$$

Hence, from L' Hospital's Rule, we have

$$\log_e y = \underset{x\to 0}{Lt} \frac{1}{\sin x \cdot -\sin x + \cos x \cdot \cos x}$$

$$= \underset{x\to 0}{Lt} \frac{-1}{-\sin^2 x + \cos^2 x} = \underset{x\to 0}{Lt} -\frac{1}{\cos^2 x - \sin^2 x}$$

$$= -\frac{\cancel{1}}{(1)^2 - 0} = -1$$

$$\therefore \log_e y = -1$$

$$\therefore y = e^{-1} = \frac{1}{e} \text{ Ans.}$$

(7) Evaluate $\lim_{x\to 0} (\cos x)^{\cot x}$.

Ans. → $\underset{x\to 0}{Lt} (\cos x)^{\cot x} \quad [1^\infty]$

Let $y = \underset{x\to 0}{Lt} (\cos x)^{\cot x}$

Taking log both sides, we have

$$\log_e y = \underset{x\to 0}{Lt} \cot \cdot \log \cos x \quad [\infty \times \infty]$$

$$= \underset{x\to 0}{Lt} \frac{\log \cos x}{\tan x} \quad \left[\frac{0}{0}\right]$$

Hence from L' Hospital's Rule, we have

$$\log_e y = \underset{x\to 0}{Lt} \frac{-\frac{\sin x}{\cos x}}{\sec^2 x}$$

$$\log_e^y = \underset{x \to 0}{Lt} \frac{\frac{-\sin x}{\cos x}}{\frac{1}{\cos^2 x \cos x}}$$

$$= \underset{x \to 0}{Lt} -\sin x \cdot \cos x$$

$$= -1 \times 0 = 0$$

$\therefore \log_e^y = 0$

$\therefore y = e^0 = 1$ Ans.

(52) Evaluate $\underset{x \to 0}{Lt} (\cos x)^{\cot^2 x}$.

Ans. $\rightarrow \underset{x \to 0}{Lt} (\cos x)^{\cot^2 x} \quad [(1)^\infty]$

Let $y = \underset{x \to 0}{Lt} (\cos x)^{\cot^2 x}$

Taking log both sides, we have.

$$\log_e^y = \underset{x \to 0}{Lt} \cot^2 x \cdot \log(\cos x) \quad [\infty \times \infty]$$

$$= \underset{x \to 0}{Lt} \frac{\log \cos x}{\tan^2 x} \quad \left[\frac{0}{0}\right]$$

Hence, from L' Hospital's Rule, we have

$$\log_e y = \underset{x \to 0}{Lt} \frac{\frac{1}{\cos x} \times (-\sin x)}{2\tan x \cdot \sec^2 x}$$

$$= \underset{x \to 0}{Lt} \frac{-\frac{\cancel{\sin x}}{\cos x}}{2\frac{\cancel{\sin x}}{\cancel{\cos x}} \cdot \frac{1}{\cos^2 x}}$$

$$= \underset{x \to 0}{Lt} -\frac{1}{2}\cos^2 x$$

$$= -\frac{1}{2} \times (1)^2 = -\frac{1}{2}$$

$$\therefore \log_e y = -\frac{1}{2}$$

$$\therefore y = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}} \text{ Ans.}$$

(53) Find $\underset{x \to 0}{Lt} (\cos x)^{1/x^2}$.

Ans.→ $\underset{x \to 0}{Lt} (\cos x)^{1/x^2} \quad [1^\infty]$

Let $y = \underset{x \to 0}{Lt} (\cos x)^{1/x^2}$

Taking log both sides, we have

$$\log_e y = \underset{x \to 0}{Lt} \frac{1}{x^2} \cdot \log \cos x$$

$$= \underset{x \to 0}{Lt} \frac{\log \cos x}{x^2} \quad \left[\frac{0}{0}\right]$$

Hence from, L' Hospital's Rule, we have

$$\log_e y = \underset{x\to 0}{Lt} \frac{\frac{1}{\cos x}\times(-\sin x)}{2\tan x \cdot \sec^2 x}$$

$$= \underset{x\to 0}{Lt} \frac{-\frac{\cancel{\sin x}}{\cos x}}{2\frac{\cancel{\sin x}}{\cancel{\cos x}}\cdot\frac{1}{\cos^2 x}}$$

$$= \underset{x\to 0}{Lt} -\frac{1}{2}\cos^2 x$$

$$= -\frac{1}{2}\times(1)^2 = -\frac{1}{2}$$

$\therefore \log_e y = -\frac{1}{2}$

$\therefore y = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$ Ans.

(53) Find $\underset{x\to 0}{Lt} (\cos x)^{1/x^2}$.

Ans. → $\underset{x\to 0}{Lt} (\cos x)^{1/x^2}$ $[1^\infty]$

Let $y = \underset{x\to 0}{Lt} (\cos x)^{1/x^2}$

Taking log both sides, we have

$$\log_e y = \underset{x\to 0}{Lt} \frac{1}{x^2}\cdot \log \cos x$$

$$= \underset{x\to 0}{Lt} \frac{\log \cos x}{x^2} \left[\frac{0}{0}\right]$$

Hence from, L' Hospital's Rule, we have

$$\log_e^y = \underset{x\to 0}{Lt} \frac{-\tan x}{2x}$$

$$= \underset{x\to 0}{Lt} \frac{-\sec^2 x}{2\times 1}$$

$$= \frac{-(1)^2}{2} = -\frac{1}{2}$$

$$\therefore \log_e^y = -\frac{1}{2}$$

$$\therefore y = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}} \text{ Ans.}$$